The Resistance of a Path

1 LAB DESCRIPTION

This lab explores the effect that geometry has on the resistance of a sample.

2 LEARNING OBJECTIVES

At the conclusion of this activity you should be able to:

• Explain the difference between electrical resistance and resistivity.
• Measure the effect of geometry on a sample's resistance to electrical current.
• Measure the resistivity of a sample.
• Calculate a weighted average.

3 BACKGROUND

3.1 WEIGHTED AVERAGE

A weighted average is a statistical tool used to combine multiple independent measurements and their uncertainties.

Weighted averages are explained in Chapter 7 (pp. 173-177) of Taylor's *An Introduction to Error Analysis*[1]. Pay particular attention to Equations 7.10, 7.11, and 7.12 and the Example given in Chapter 7.3.

3.2 INTRINSIC AND EXTRINSIC PROPERTIES

Physical properties of an object can be categorized as either *intrinsic* or *extrinsic*. Intrinsic properties typically depend on the chemical composition of an object and do not depend on the amount or shape of the sample being measured. An example of an intrinsic property of a material is density. Extrinsic properties are not essential - or inherent properties of a material. Extrinsic properties depend on the amount or shape of the sample. An example of an extrinsic property is weight.
The resistivity (typically designated $\rho$) of an object is an intrinsic property that describes a material's opposition to an electric current. Conductors are materials that have a very low resistivity while insulators have high resistivity.

The resistance (typically designated $R$) of an object is an extrinsic property that depends on the resistivity of the material and on the specific geometry of the sample.

3.3 Bulk Resistance

The resistance of a sample depends on the intrinsic resistivity $\rho$ and the extrinsic geometric properties of the sample\(^2\). The total resistance of a sample is given by:

$$R = \rho \frac{l}{A}, \quad (3.1)$$

where $l$ is the length of the sample and $A$ is the cross-sectional area that is given by the height $h$ and width $w$. The resistivity $\rho$ is measured in units of resistance times length ($\Omega \cdot m$ in SI units).

![Diagram of a sample of resistivity $\rho$ ($\Omega \cdot m$) with length $l$ and cross-sectional area $A$ ($A = h \cdot w$). The sample is electrically connected to conductors with negligible resistance at either end. The total resistance of the sample depends on the geometry: the resistance is proportional to the length and inversely proportional to the cross-sectional area.](image)
3.4 Sheet Resistance

Sheet resistance is a special case of bulk resistance that is used to characterize the resistance of materials that are uniform thin sheets[3]. Imagine a thin sheet of with thickness $h$, length $l$, and width $w$ like the one shown in Figure 3.2. Since this is a thin sheet, $h$ is much smaller than $l$ and $w$. A new quantity is defined to describe the surface resistivity $R_s$ of a thin film:

$$R_s = \frac{\rho}{h}, \quad (3.2)$$

where $\rho$ is traditional resistivity and $h$ is the thickness of the film. Note that the units of $R_s$ are resistance times length divided by length ($\Omega \cdot m/m$ or simply $\Omega$ in SI units). Surface resistivity and resistance both have the same measurement units. In order to avoid confusion, surface resistivity is typically expressed in units of “ohms per square” ($\Omega /$sq, or $\Omega /\square$).

The resistance of a thin film is given by:

$$R = R_s \frac{l}{w}. \quad (3.3)$$

Figure 3.2: A diagram of a thin film sample with surface resistivity of $R_s (\Omega /$sq) (shown in white) with thickness $h$, width $w$, and length $l$ where $h \ll w, l$. The sample is electrically connected to conductors with negligible resistance (shown in gray) at either end. The total resistance of the sample depends on the geometry; the resistance is proportional to the length and inversely proportional to the width.

3.5 Resistors in Series and Parallel

Multiple resistors can either be connected in series (Figure 3.3a) or parallel (Figure 3.3b). The total resistance $R_T$ for $n$ resistors connected in series[4] is given by:

$$R_T \text{ (series)} = \sum_n R_n. \quad (3.4)$$

The total resistance for $n$ resistors connected in parallel[5] is given by:

$$\frac{1}{R_T \text{ (parallel)}} = \sum_n \frac{1}{R_n}. \quad (3.5)$$
Figure 3.3: Examples of resistors connected in series and parallel. The total resistance in the series circuit is given by $R_{T\text{ (series)}} = R_1 + R_2$. The total resistance of the resistors connected in parallel is given by: $1/R_{T\text{ (parallel)}} = 1/R_1 + 1/R_2$ which simplifies to $R_{T\text{ (parallel)}} = (R_1 R_2)/(R_1 + R_2)$.

4 Procedure

The goal of this lab is to measure the surface resistivity $R_s$ of a thin film sample. The sample in question is a sheet of paper made to conduct electricity by the inclusion of a carbon filler that has a low resistivity.

Model the surface resistivity using Equation 3.3 like you did in the pre-lab quiz exercises.

Use scissors to modify the shape of your paper samples and “bulldog” paper clips to make electrical connections (like those shown in Figure 3.2). There are two modifiable dimension parameters, length and width. In each of your experiments, decide which parameter will be held constant and which will be varied.

One of the main challenges of measuring the electrical properties of a material is making reliable electrical connections. Think carefully about how to best connect to your sample and how the size of the sample might affect your measurement.
5  LAB NOTE

Write a brief lab note to record and communicate your work. Limit your note to about 1000 words. Your note should be concise and to the point. Bullet points are encouraged.

Please include pictures, figures, diagrams, etc to clearly explain your ideas. Equations can be included using an equation editor or a photo of your handwritten work.

Your note should address the following general ideas:

- **Experiment Purpose (4 points)**
  - In your own words, state the purpose of this experiment.

- **Calculations & Error Propagation (6 points)**
  - Present the equations used to analyze your collected data.
  - Present the equations used to account for and propagate error.

- **Data Collection & Analysis (6 points)**
  - *Briefly describe the method – beyond what was already stated in the assignment – you used to accurately collect your data.*
  - Measured quantities – that do not appear directly in your plot(s) – should be presented in an organized table.
  - Analyze your data using well-formatted plot(s). Clearly show how the plot is used to arrive at your result(s).

- **Results (4 points)**
  - Clearly state the final result(s) of your experiment. Remember to quote your result with units and appropriate significant digits.

- **Discussion**
  - **Observations (4 points)**
    * What did you observe in your experiment?
    * Relate your observations to your understanding of the underlying physics.
  - **Significance (4 points)**
    * Compare your measured results to each other and/or to a known/expected value.
    * Choose the best tools for your comparison (e.g. plots, pictures, discrepancy, significance, etc).
  - **Confidence (8 points)**
    * The goal is to communicate to your audience how seriously your result should be taken.
    * Discuss how confident you are in your result.
    * Discuss factors that may be affecting the *accuracy* and *precision* of your result.
    * Suggest improvements to your experiment to address your accuracy and precision.
• Style (4 points)
  – Points are allocated to notes that follow the guidelines for the assignment.
  – Examples of things that fall under the “style” category include (but are not limited to): word count, formatting, attention to activity-specific instruction, readability, creativity, etc.

REFERENCES


[5] Ibid. See Chapter 9.7 (pp. 103-104).