Conservation of Momentum: Inelastic Collisions

Conservation laws are some of the most powerful concepts in physics. In this lab we will explore the conservation of momentum and energy by studying *inelastic* collisions.

1 LEARNING OBJECTIVES

At the conclusion of this activity you should be able to:

- Identify the difference between elastic and inelastic collisions.
- Design an experiment to measure momentum and energy conservation.
- Use a photogate to measure the velocity of an object.

2 BACKGROUND

Conservation laws can be used to describe complicated physical systems. For example, collisions involving fundamental particles can be accurately reconstructed using the conservation of energy and momentum. At larger scales, conservation laws allow us to describe stellar and galactic dynamics.

Conservation laws require that the initial and final quantities of an observable are the same. The conservation of energy and momentum are often useful for solving many problems in general physics.

2.1 CONSERVATION OF MOMENTUM

Recall that the momentum[1] of a system is given by:

\[ \vec{p} = m \vec{v}, \]  

(2.1)

where \( m \) is the mass of an object and \( \vec{v} \) is the velocity.

The total momentum \( \vec{P}_T \) of a system of \( n \) discrete parts can be expressed as the sum of each of the parts:

\[ \vec{P}_T = m_1 \vec{v}_1 + m_2 \vec{v}_2 + \cdots + m_n \vec{v}_n \]  

(2.2)

When the sum of external forces acting on a system is zero, momentum is conserved. That is, the total initial momentum of the system \( \vec{P}_I \) is equal to the total final momentum \( \vec{P}_F \):

\[ \vec{P}_I = \vec{P}_F. \]  

(2.3)
2.2 Kinetic Energy

An object of mass \( m \) and velocity \( \vec{v} \) has a kinetic energy\(^{[2]} \) given by:

\[
K = \frac{1}{2} m \vec{v}^2
\]  

(2.4)

The total kinetic energy \( K_T \) of a system of \( n \) discrete parts can be expressed as the sum of each of the parts:

\[
K_T = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \cdots + \frac{1}{2} m_n v_n^2.
\]

(2.5)

2.3 Elastic Collisions

Elastic collisions\(^{[3]} \) involve only conservative forces. In these collisions the total kinetic energy of the objects involved is conserved:

\[
K_i = K_f,
\]

where \( K_i \) and \( K_f \) are the total initial and final kinetic energies of the system respectively.

In an elastic collision, the total initial kinetic energy (sum of the parts) is equal to the total final kinetic energy. Examples of (nearly perfect) elastic collisions are those between billiard balls.

2.4 Inelastic Collisions

Inelastic collisions do not conserve energy\(^{[4]} \). The total initial kinetic energy of the system is not equal to the total final kinetic energy:

\[
K_i \neq K_f
\]

(2.7)

where \( K_i \) and \( K_f \) are the total initial and final kinetic energies respectively.

An example of an inelastic collision is a car crash. Automobiles are designed to collapse and deform to absorb as much energy as possible (maximize the value of \( F \)) in a collision to protect the passengers.

2.5 Fractional Energy Loss

Fractional energy loss \( F \) is one way to quantify the amount of energy “lost”\(^{[5]} \) in a collision. The fractional energy loss is given by:

\[
F = \frac{K_i - K_f}{K_i}
\]

(2.8)

where \( K_i \) is the total kinetic energy of the system \textit{before} the collision and \( K_f \) is the total kinetic energy of the system \textit{after} the collision.

In a perfectly elastic collision, the fractional energy loss is equal to zero.
Suppose we have two masses \( m_2 \) and \( m_1 \) that collide. Before the collision, \( m_2 \) is initially at rest and \( m_1 \) has velocity \( v_i \). After the collision, the two masses stick together and travel with velocity \( v_f \). In this specific case, the fractional energy loss can be written in terms of the ratio of initial and final velocities.

\[
F = \frac{K_i - K_f}{K_i} \tag{2.9}
\]

\[
F = \frac{\frac{1}{2} m_1 v_i^2 - \frac{1}{2} (m_1 + m_2) v_f^2}{\frac{1}{2} m_1 v_i^2} \tag{2.10}
\]

\[
F = 1 - \left( \frac{m_1 + m_2}{m_1} \right) \left( \frac{v_f}{v_i} \right)^2 \tag{2.11}
\]

### 3 Procedure

You will use gliders on an (approximately frictionless) air track\[6\] to study inelastic collisions.

The goal is to measure the fractional energy loss \( F \) during inelastic collisions between carts of different masses.

The velocity of each cart can be measured directly using Logger Pro and photogates. A photogate is an optical device that is used to accurately time events. When an object obstructs the beam of a photogate the timer starts, when the obstruction is removed, the timer stops. Based on the dimensions of the object that obstructs the photogate and the measured time, a velocity can be calculated. For more information on the photogate you are using see: [https://www.pasco.com/file_downloads/Downloads_Manuals/Photogate-Head-Manual-ME-9498A.pdf](https://www.pasco.com/file_downloads/Downloads_Manuals/Photogate-Head-Manual-ME-9498A.pdf)

Most of the work of photogate timing is handled by the Logger Pro DAQ software. In Logger Pro, open the file: `File ▸ Open... ▸ Probes & Sensors ▸ Photogates ▸ Collision Timer.cml`. This Logger Pro configuration file will be used throughout the experiment.

#### 3.1 Experiment

- Setup inelastic collisions with a moving glider (Cart 1 with mass \( m_1 \)) colliding with a stationary glider (Cart 2 with mass \( m_2 \)). Vary \( m_1 \) and \( m_2 \) by adding additional masses to the carts. Note that mass must be added symmetrically to each cart to keep the cart balanced on the air track.

- For each mass configuration:
  - Compute the predicted fractional energy loss using the equation you derived in the Pre-lab Quiz that depends only on the cart masses.
  - Measure the initial velocity of cart 1 \( v_i \) and the final velocity of the cart 1 + cart 2 system \( v_f \) for several (at least 5) collisions. Calculate the average ratio of velocities \( \frac{v_f}{v_i} \). Use statistics to estimate the uncertainty associated with the average ratio of velocities \( \sigma_{\frac{v_f}{v_i}} \).
  - Use \( \frac{v_f}{v_i} \) and Equation 2.11 to obtain the observed fractional energy loss.
  - Use error propagation to obtain the uncertainty associated with the fractional energy loss for the series of collisions.
• Repeat the above steps for several different \( m_1 \) and \( m_2 \) mass configurations.

3.2 SUGGESTIONS FOR ANALYSIS

• Visualize the collected data by making the plot described in the Pre-lab Quiz.
• Use \texttt{LINEST()} to quantitatively describe the relationship between the observed and predicted fractional energy losses.

4 LAB REFLECTION

Write a brief reflection to document and summarize your lab work.

Your work will be evaluated using the following rubric:

• Data Analysis & Plots (4 points)
  – State what you intend to accomplish with your experiment. What are you trying to measure?
  – Briefly describe how the apparatus will be used to make the measurement. A well-labeled photograph or diagram is an efficient way to explain the details of the experimental setup.
  – Analyze your data using well-formatted plot(s).
  – Describe the models that were used to make sense of your data.

• Result(s) and Comparison (4 points)
  – Clearly state the final result(s) of your experiment with an associated uncertainty estimate.
  – Report your result with measurement units and the appropriate significant digits.
  – Compare your result to another relevant quantity. For example, you might compare your measurement to an accepted value or another another group's value.
  – Choose the best tools available to make the comparison meaningful.

• Dominant Source of Uncertainty (4 points)
  – Identify and discuss the dominant source(s) of uncertainty in your result.
  – Use error propagation calculations (as appropriate) to support your explanation.

• Experiment Reflection (4 points)
  – Interpret the evidence (plots, results, calculations, error estimates, etc) that you have presented.
  – Explain how your experimental findings relate to the underlying physical principles.
  – Emphasize interesting features of your experiment and/or highlight unanswered questions that you identified in the course of the experiment.
  – Remember that we are interested in the details of your experiment and not vague theoretical statements.
REFERENCES


[2] Ibid. See Unit 7.2 (pp. 72-73).

[3] Ibid. See Units 11.5 (pp. 124-125) and 12.2-12.3 (pp. 132-135).


[5] Ibid. See Unit 11.6 (pp. 125-126).